Permanent displacement of rock slope considering degradation of slide surface during earthquake

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Abstract
A new approach for evaluating the permanent displacement including planar block slide and wedge block slide of rock mass slope during earthquake is presented, which takes degradation of sliding surface (friction weakening with sliding velocity and displacement) during earthquake into account. The flow for putting the method into effect is put forward, and three cases are studied. The results indicate that the approach is available and reasonable to assess the value of the permanent displacement of rock mass slope. The result reached by traditional Newmark method(1965) is too small in general compare to the lab test result. Traditional Newmark method(1965) is only available to evaluating the rock mass slope with small permanent displacement during earthquake.

Keywords: permanent displacement, rock mass slope, degradation of slide surface, Newmark method

1. Introduction
Earthquake can trigger a large number of earth and or rock slides, some of which are capable of causing deaths and major damages to major structures, important roadways, dam reservoir etc. For example, more than 15000 landslides induced by 2008 Wenchuan 7.9 Mw earthquake, claimed deaths over 20,000, damaged road 220,000 km and 2,900 bridges (Yin et al., 2009; Qi et al, 2010).

Newmark (1965) has suggested a famous approach to obtain the earthquake induced displacement suitable for rigid plastic materials in the Fifth Rankine Lecture. He realized that whether the artificial earth fill dam was stable or not depended on the deformation during earthquake, rather than traditional slope safety factor less than 1.0 and what related to the seismic deformation directly was the changes of the stress history, rather than the maximum stress. This approach has been used successfully to predict the surface displacements of banks of dry, cohesionless soils subjected to known series of base motions (Goodman and Seed, 1966; Bustamente, 1965). However, the method assumes that yield acceleration is constant and the material is rigid, which does not meet the real behavior of rock mass.

The dams filled with loose sand and moderate-dense saturated sandy soil bring about serious liquefaction and the yield acceleration decreased obviously accompanied with the accumulation of pore pressure during earthquake. For this reason, many attempts were made to improve this model, resulting in models for obtaining the earthquake induced displacement (Franklin and Chang, 1977; Makdisi and Seed, 1978; Sarma, 1975; Sarma, 1981; Seed et al., 1969; Seed, 1979; Richar and Elms, 1979; Lin and Whitman, 1986; Nadim and Whitman, 1983; Kramer and Smith, 1997; Rathje and Brady, 1999). Among them, Seed-Lee-Idriss method proposed by Seed in the 19th Rankine Lecture is the most representative (Seed, 1979).

Rock slope dynamic response is controlled by discontinuities. Hendron (1971) introduced Newmark method (Newmark, 1965) to predict permanent displacement of rock block during earthquake. Crawford (1981, 1982) did dynamic test of the rock mass and found for a flat structural surface that the earthquake yield acceleration changes with the cumulative displacement and rate of the rock movement, rather than the constant. On this basis, the calculating method of earthquake permanent displacement of planar rock block was put forward, which took the cumulative displacement and rate of the rock movement of the discontinuities seismic yield acceleration into consideration. For a granite
block placed on a flat plane in lab test, Wang and Zhang (1982) found that the dynamic friction coefficient is lessening gradually with increasing of the displacement and rate of the block movement. Based on this, they put forward a method to evaluate the permanent displacement of the planar rock block.

Discontinuities are not smooth but fluctuant in the rock mass (Patton, 1966). Dynamic tests of the rock mass showed that the discontinuities deteriorated gradually under the cycle load (Hutson, 1987; Plesha, 1987; Hutson and Dowding, 1990; Jing et al., 1993; Kana et al., 1996; Fox et al., 1998; Lee et al., 2001; Homand et al., 2001; Yang et al., 2001), the asperities of the discontinuities plane reduced and undulation angles deteriorated gradually under the cycle load (Hutson, 1987; Plesha, 1987; Hutson and Dowding, 1990; Jing et al., 1993; Kana et al., 1996; Fox et al., 1998; Lee et al., 2001; Homand et al., 2001; Yang et al., 2001), the asperities of the discontinuities plane reduced and undulation angles deteriorated gradually. The asperities of discontinuities are closely related to the shear strength of surface (Barton, 1971, 1973, 1976), thus the degradation of the discontinuities would affect the permanent displacement of rock mass slope directly.

This paper presents a new approach to evaluate the permanent displacement of rock slope including planar block slide and wedge block slide, which considers the degradation of asperities of sliding surface during earthquake. The procedure of the method is put forward and some cases are illustrated to proof its effectiveness.

2. Rock slope model

2.1 Rock block slide model

Fig. 1 is the rock block slope model which assumes that the mass of slide block is \( m \), the dip angle of the slide surface is \( \theta \) and the seismic acceleration is \( Ag \) where \( A \) is seismic coefficient. The acute angle formed by seismic force and horizontal line is \( \beta \). The coordinate system is shown in Fig. 1.

![Fig. 1 Rock block slope model](image)

Considering the equilibrium conditions of force in Y direction, we can get

\[
N = mg \cos \beta + mAg \sin(\theta - \beta) \tag{1}
\]

Where \( N \) is supportive force on which the slide plane gives the slide.

In the X direction, we can have

\[
mAg \cos(\theta - \beta) + mgsin\beta - N \tan \phi = ma \tag{2}
\]

where \( \phi \) is the angle of internal friction, \( x \) is the displacement of the slide block, \( a \) is the acceleration of the slide block. Then we can get

\[
a = Ag \cos(\theta - \beta) + g \sin \beta - [g \cos \beta + A \sin(\theta - \beta)] \tan \phi \tag{3}
\]

When \( a = 0 \), \( A \) becomes the yield acceleration coefficient, \( A_y \):

\[
A_y = \frac{(\cos \beta \tan \phi - \sin \beta) / [\cos(\theta - \beta) - \sin(\theta - \beta)] \tan \phi}{\sin(\beta - \phi) / \cos(\theta - \beta + \phi)} \tag{4}
\]

\[
a = \frac{[Ag \cos(\theta - \beta + \phi) + g \sin \beta - [g \cos \beta + A \sin(\theta - \beta)]] \tan \phi}{\cos \phi} = \frac{g \cos(\theta - \beta + \phi)(A - A_y) / \cos \phi}{\cos \phi} \tag{5}
\]

where,

\[
\phi = \phi_b + \alpha_k \tag{6}
\]

\( \phi_b \) is the basic friction angle of the slide surface, \( \alpha_k \) is the first order asperity angle of the slide surface (Patton, 1966), as shown in Fig. 2.

![Fig. 2 Conception model of one and two grades asperity angle (Patton, 1966)](image)

Substituting Eq. (6) into Eq. (5) gives

\[
a = Ag \cos(\theta - \beta) + g \sin \beta - [g \cos \beta + A \sin(\theta - \beta)] \tan(\phi_b + \alpha_k) = Ag \cos(\theta - \beta) + g \sin \beta - [g \cos \beta + A \sin(\theta - \beta)] \tan(\phi_b + \tan \alpha_k) / [1 - \tan \phi_b \tan \alpha_k] \tag{7}
\]

For flat slide surface \( \alpha_k = 0 \), then \( \phi = \phi_b \). Assume that \( a = 0 \), and we can get

\[
\tan \phi_b = \frac{[\cos(\theta - \beta)A_y + \sin(\beta / \cos \beta)] / [\cos \beta + \sin(\theta - \beta)A_y]}{[\cos \beta + \sin(\theta - \beta)A_y]} \tag{8}
\]

2.2 Wedge block slide model
Wedge block slide failure is a common failure mode for rock slope. Static problem for wedge block slide was discussed by Hoek and Brown (1977).

Fig. 3 is a wedge block formed by two penetrative discontinuities JA and JB of rock slope. Assuming that under a seismic acceleration \( A \), the wedge block only vibrates in the vertical plane crossing the intersection line of the two discontinuities. The angle of inclination of intersection line is \( \beta \), the mass of wedge is \( m \), the normal force on JA and JB are \( R_A \) and \( R_B \), both the internal friction angle of two discontinuities are \( \phi \). The coordinate system is shown in Fig. 3.

In the X direction, we can have

\[
mgsin\beta + mAgcos(\theta - \beta) - (R_A + R_B)tan \phi = ma \quad (9)
\]

Consider equilibrium conditions of the plane normal to the intersection line of two discontinuities, and assume \( N \) is component of mass force on this plane, we can get

\[
N = mgcossin\beta + mAgsin(\theta - \beta) \quad (10)
\]

According to the equilibrium conditions, we can get

\[
R_A = \frac{[Nsin(\delta + \gamma)]}{sin2\gamma} \quad R_B = \frac{[Nsin(\delta - \gamma)]}{sin2\gamma}
\]

Thus,

\[
R_A + R_B = \frac{Nsin\delta}{siny} = \frac{mgsin\delta[cos\beta + Asin(\theta - \beta)]}{siny}
\]

Substituting Eq. (12) into Eq. (9) gives

\[
a = gsin\beta + Agcos(\theta - \beta) - \frac{gsin\delta[cos\beta + Asin(\theta - \beta)]tan\phi}{siny} \quad (13)
\]

Substituting Eq. (6) into Eq. (13) gives

\[
a = gsin\beta + Agcos(\theta - \beta) - \frac{gsin\delta[cos\beta + Asin(\theta - \beta)]tan\phi}{siny} \quad \frac{1}{1 + tan\phi_k tan\alpha_k} \quad (14)
\]

Assuming that \( a = 0 \) and \( \alpha_k = 0 \), the Eq. (13) can be expressed

\[
\tan \phi_k = \frac{[\sin b / \sin y + A_k \cos(\theta - \beta) \sin y]}{[\sin \phi b + A_k \sin(\theta - \beta) \sin y]} \quad (15)
\]

3. Approach to evaluate permanent displacement considering degradation of slide surface during earthquake

3.1 Law of degradation of the asperity angle (\( \alpha_k \)) of slide surface

Under the role of the cyclic shear, asperities of the slide surface are deteriorated, as \( \alpha_k \) decreased gradually. According to Plesha (1987), we can have

\[
\alpha_k = (\alpha_k)_0 \exp(-cWp) \quad (16)
\]

Where \( \alpha_k_0 \) is the initial asperity angle, \( c \) is the joint damage coefficient \((m^2/J)\) that is a test constant and \( Wp \) is the plastic work.

According to Hutson (1987) and Hutson and Dowding (1990),

\[
c = 0.114JRC(\sigma_n / \sigma_c) \quad (17)
\]

where \( \sigma_c \) is the uniaxial compressive strength of the material, \( \sigma_n \) is the normal force of the sliding surface during the shearing process.

3.2 Law of degradation of the basic friction angle (\( \phi_k \)) of slide surface

For flat discontinuities, the asperity angle (\( \alpha_k \)) equals to 0, and the yield acceleration \( A_c \) is dependent on the basic friction angle of \( \phi_k \).

Through extensive researches, Crawford and Curran (1982) found that for the flat discontinuities the yield acceleration \( A_c \) was the function of the cumulative displacement \( x \) and displacement rate \( v \). Thus it can be expressed as \( A_c(x, v) \). For the effect of cumulative displacement, Crawford and Curran (1982) adopted the model of post-peak constant displacement which is rigid and strain-softening to describe as

\[
A_c(x) = A_m[1 - (1 - p)(x / x_0)], x \leq x_0 \\
A_c(x) = A_m p, x > x_0 \quad (18)
\]

where \( p \) is the reduction coefficient which can be determined by tests, \( A_m \) is the maximum acceleration of input seismic waves. According to extensive researches, Crawford and Curran (1982) believed that \( x_0 \) takes 50mm appropriately for most of the rock.

According to research of Crawford and Curran (1981, 1982), the rate effect of yield acceleration \( A_c(v) \) is...
can be expressed as

\[ A_\alpha(v) = A_m[1.0 \pm 0.20\log(v / \nu_0)], v \geq \nu_0 \]
\[ A_\alpha(v) = A_m, v < \nu_0 \]  

(19)

Where “+” stands for different material. For weak materials (such as dolomite), take “−” while for hard materials (such as black granite, sandstone) take “+”. Crawford and Curran (1982) found that \( \nu_0 \) is usually 10mm/s through abundant test results.

If not consider cumulative displacement effect and rate effect, we can get \( A_\alpha(x, v) = A_m \) according to Eq. (18) and Eq. (19). That means that yield acceleration is equal to maximum seismic acceleration, and this is obviously unreasonable. Eq. (18) and Eq. (19) proposed by Crawford and Curran (1982) have conceptual errors, and \( A_m \) should be changed into initial acceleration \( A(0, 0) \). Meanwhile, Crawford and Curran (1982) did not consider about cumulative displacement effect and rate effect for Eq. (18) and Eq. (19) at the same time. Therefore, the paper considers initially the rate effect and revised Eq. (18) as

\[ A_\alpha(v, x) = A_\alpha(0, 0)[1.0 \pm 0.20\log(v / \nu_0)], v \geq \nu_0 \]
\[ A_\alpha(v, x) = A(0, 0), v < \nu_0 \]  

(20)

Then considering the effects of cumulative displacement again, Eq. (20) can be expressed as

\[ A_\alpha(v, x) = pA_\alpha(0, 0), x > 50\text{mm}, v < 10\text{mm/s} \]
\[ A_\alpha(v, x) = A_\alpha(0, 0)[1 - (1 - p)(x / 50)], x \leq 50\text{mm}, v < 10\text{mm/s} \]
\[ A_\alpha(v, x) = pA_\alpha(0, 0)[1 \pm 0.20\log(v / 10)], x > 50\text{mm}, v \geq 10\text{mm/s} \]
\[ A_\alpha(v, x) = A_\alpha(0, 0)[1 - (1 - p)(x / 50)][1 \pm 0.20\log(v / 10)], x \leq 50\text{mm}, v \geq 10\text{mm/s} \]  

(21)

Thus substituting Eq. (21) into Eq. (8) and Eq. (15), we can get the law of degradation of the basic friction angle (\( \phi_b \)) of slide surface for models shown in Fig. 1 and Fig. 3 respectively.

Substituting Eq. (16), Eq. (17), Eq. (21) and Eq. (8) into Eq. (7) gives acceleration time travel history of the slide. By double integration, we can get seismic permanent displacement of planar sliding block as shown in Fig. 1 considering the degradation process of slide surface during earthquake.

Substituting Eq. (16), Eq. (17) and Eq. (21) and Eq. (15) into Eq. (14), we can obtain motion acceleration of wedge under dynamic load. By double integration, we can obtain seismic permanent displacement of rock wedge sliding block as shown in Fig. 3 considering the degradation process of the slide surface during earthquake.

4. Procedures for calculating seismic permanent displacement of rock slope

(1) Substitute initial asperity angle of sliding surface (\( \alpha_0 \)) and initial basic friction angle of sliding surface (\( \phi_b \)) into Eq. (7) or Eq. (14) assume \( a = 0 \) we can get \( A(0, 0) \) of the block in Fig. 1 or Fig. 3;

(2) Discretize acceleration time-travel history into \( n\Delta t \) as \( A(0), A(\Delta t), A(2\Delta t), \ldots, A(n\Delta t) \).

(3) Substitute \( A(0) \), initial asperity angle of sliding surface (\( \alpha_0 \)), initial basic friction angle of sliding surface (\( \phi_b \)) and \( A(0, 0) \) into Eq. (7) or Eq. (14), we can get (\( \alpha_0 \)), and then figure out (\( v \)). If (\( v \)) > 0, continue next step (4); if not, forward timestep, update corresponded time history of \( A(t) \), and continue step (3) till (\( v \)) ≥ 0;

(4) Integrate and obtain m1 steps’ displacements of (\( v \))m;

(5) According to displacement (\( v \))m, we can get the work done by slide force and obtain the deteriorated asperity angle of sliding surface (\( \alpha_0 \))m;

(6) Substituting (\( v \))m and (\( v \))m into Eq. (21) and Eq. (8) or Eq. (15) gives tan(\( \phi_b \))m;

(7) Substituting (\( \alpha_0 \))m and (\( \phi_b \))m into Eq. (7) or Eq. (14) gives m1 steps’ acceleration (\( a_0 \))m, and then figure out (\( v \))(m+1). If (\( v \))(m+1) ≥ 0, continue step (4); if not, forward timestep, update corresponded time history of \( A(t) \), and continue step (3) till (\( v \))(m+n) ≥ 0;

(8) Repeating the above steps (3)-(7) till the n step, we can get the slope seismic permanent displacements (\( x \))n = (\( x \))(n-1) + (\( v \))(n)\Delta t.

5. Cases

(1) Case 1

The case is shown as Fig. 1 and is from Wang and Zhang (1982), where \( \beta = 20^\circ \), vibration direction is horizontal (\( \theta = 0 \)), acceleration time travel history of bedrock is shown in Fig. 4 with maximum acceleration of 10.20m/s², sliding surface is flat and without asperity (\( \alpha_0 = 0 \)), tan(\( \phi_b \)) = 0.75 and time step \( \Delta t = 0.001s \). The lithology of the bed rock is granite.

Take \( p = 1 \) (not considering the deteriorated effect of the sliding surface ), \( p = 0.65 \) and \( p = 0.70 \) in Eq.
(21) respectively, and use procedures as detailed in section 4, we can evaluate the permanent displacement of the block as shown in Fig. 5. As it can be seen in the figure, the result reached based on $p = 0.65$ (805 mm) or $p = 0.70$ (638 mm) is much closer the lab tested result (730 mm) given by Wang and Zhang (1982). While the result calculated by Newmark (1965) or the result reached based on $p = 1$ is much less that the tested result (see Fig. 5). The traditional Newmark method (1965) which didn’t consider the deteriorated effect of the sliding surface will give much less estimated seismic permanent displacement of the slope.

Fig. 5 Comparison of permanent displacement of case 1 reached by different measures

(2) Case 2
The case is identical to case 1 except for $\beta = 30^\circ$, and the calculated result is shown in Fig. 6.

Fig. 6 Comparison of permanent displacement of case 2 reached by different measures

As it can be seen from Fig. 6, the result reached based on $p = 0.1$ is much closer to the tested result given by Wang and Zhang (1982). The result of Newmark method (1965) is much smaller than the test result, and even result given based on $p = 1$, see Fig. 6.

(3) Case 3
The case is a wedge shaped block as shown in Fig. 3, where $\beta = 43^\circ$, $\delta = 70^\circ$, $\gamma = 35^\circ$, the discontinuity $J_A$ and $J_B$ have the same friction parameters with $c = 0$, $\tan(\phi_b)$, and the rock density is $2.2 \times 10^3$ kg/m$^3$. Acceleration time travel history of bedrock is shown in Fig. 7, with the peak acceleration of 0.1g, horizontal vibration direction ($\theta = 0$) and time step $\Delta t$ of 0.005s. The calculated results can refer to Fig. 8.

As it can be seen in the figure, the results of traditional Newmark method (1965) is still near 2mm less than the results ($p = 1$) of the paper. What not to consider degradation of discontinuities has significant impact on the seismic permanent displacement of the slope.

Fig. 7 Acceleration time travel history of Bedrock for case 3

Fig. 8 Comparison of permanent displacement of case 3 reached by Newmark (1965) and the approach presented in the paper

6. Discussions and conclusions

The paper puts forward the algorithm for evaluating seismic permanent displacement of the plane block sliding and wedge block sliding of the rock slope which is taken degradation of the sliding surface into consideration. Through three cases studies, the paper shows that:

(1) Three cases indicate that the degradation of the sliding surface has significant impact on seismic permanent displacement of the slope. The degradation of sliding surface must be taken into account when estimating the slope seismic permanent displacement. The method presented in the paper is able to estimate the value of slope seismic permanent displacement well, and the minimum value can be given when $p = 1.0$, while the maximum can be taken...
when $p = 0.1$.

(2) The traditional Newmark method (1965) does not consider degradation of sliding surface. The seismic permanent displacement of the slope given by this method is too small. For stable slope or one with small value of seismic slope permanent displacement, result given by the traditional Newmark method (1965) is a little closer to the result given by the method presented in the paper which considered the degradation of sliding surface. Case 3 clearly indicated this point.

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